Unit 14: Transformations (Geometry)

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Vocabulary ☺ Unit 14

image	a figure resulting from a transformation
pre-image	a figure before a transformation occurs
transformation	a change in the size or position of a figure
translation	a movement (slide) of a figure along a straight line
image	a figure resulting from a transformation
pre-image	a figure before a transformation occurs
reflection	a transformation of a figure that flips the figure across a line
clockwise	the figure moves the same direction as a clock moves
counterclockwise	the figure moves the opposite direction as a clock
origin	the point where the x-axis and y-axis intersect on the coordinate plane
rotate 180 degrees	the figure turns two quadrants
rotate 270 degrees	the figure turns three quadrants
rotate 90 degrees	the figure turns one quadrant
rotation	a transformation in which a figure is turned about a point
isometry	a transformation in which an original figure and its image are congruent
center of dilation	the point of intersection of lines through each pair of corresponding vertices in a dilation
dilation	transformation that changes the size, but not the shape, of a figure
scale factor	the ratio used to enlarge or reduce similar figures
composition of transformation	A composition of two transformations is a transformation in which a second transformation is performed on the image of a first transformation
glide reflection	A composition of a translation and a reflection in a line parallel to the direction of the translation

Translations

<u>Pre-Image</u>: The original figure before a transformation

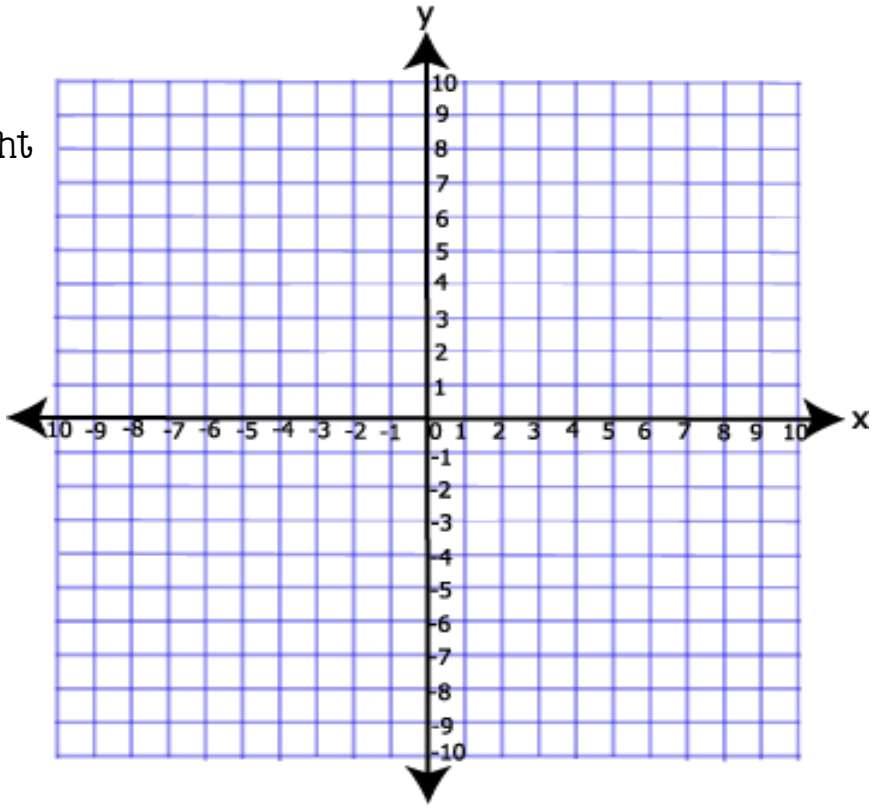
<u>Image</u>: The new figure after a transformation

***Once a point is moved to its new position it is called a "prime point" and named like this: A' (A Prime)**

<u>Translations</u>: involve moves that are either right-left, up-down, or a combination of these. Sometimes referred to as a slide.

Example 1: Create a pre-image by graphing and labeling the following points:

A(-3, 2), B(-3, 6), C(-7, 2)



Now take each point and move it 8 units right and then label the new points as "primes".

Name the new prime coordinates below:

A'(_____,_____), B'(_____,_____), C'(_____,_____)

Did the shape or size of the figure change?

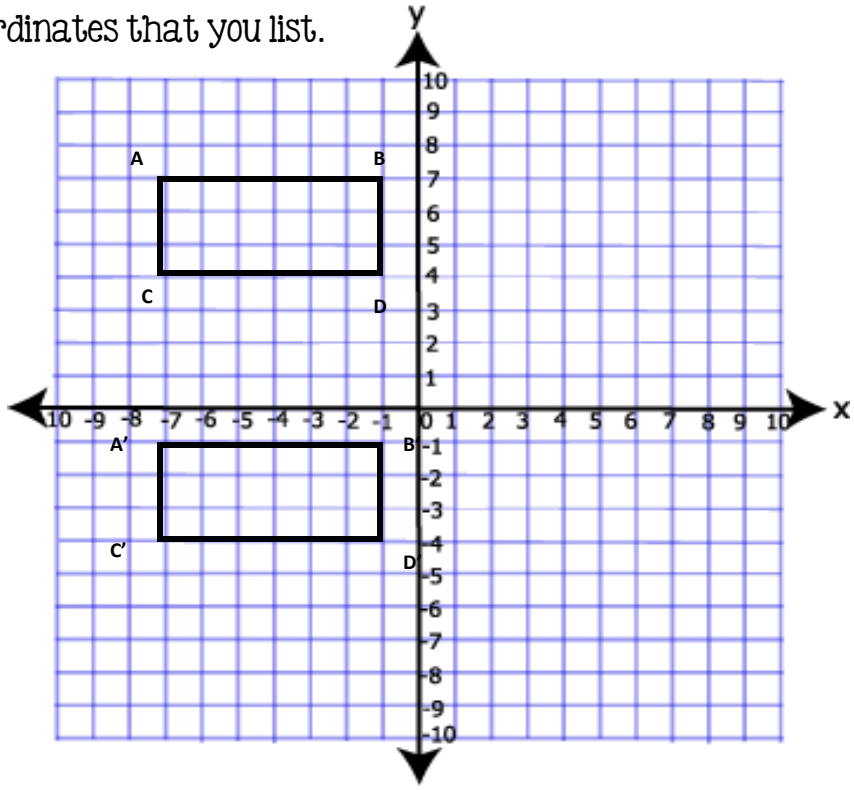
Look at the new "x" numbers. What do you notice happened to the "x" part of each coordinate?

Why do you think it was the x affected and not the y?

What type of move do you think would affect the y?

Example 2: Look at the graphed image below. Write in the coordinates for the pre-image and the image. Make sure that you label the coordinates that you list.

Pre-Image Coordinates	Image Coordinates



How did you determine which was the pre-image and which is the image?

Look at the new "y" numbers. What do you notice happened to the "y" part of each coordinate?

Why do you think it was the y affected and not the x?

Translation Rules

If you are moving UP or DOWN the _____ part of your ordered pair will change. If you go up you will _____ the number of units to the original y. If you go down you will _____ the number of units from the original y.

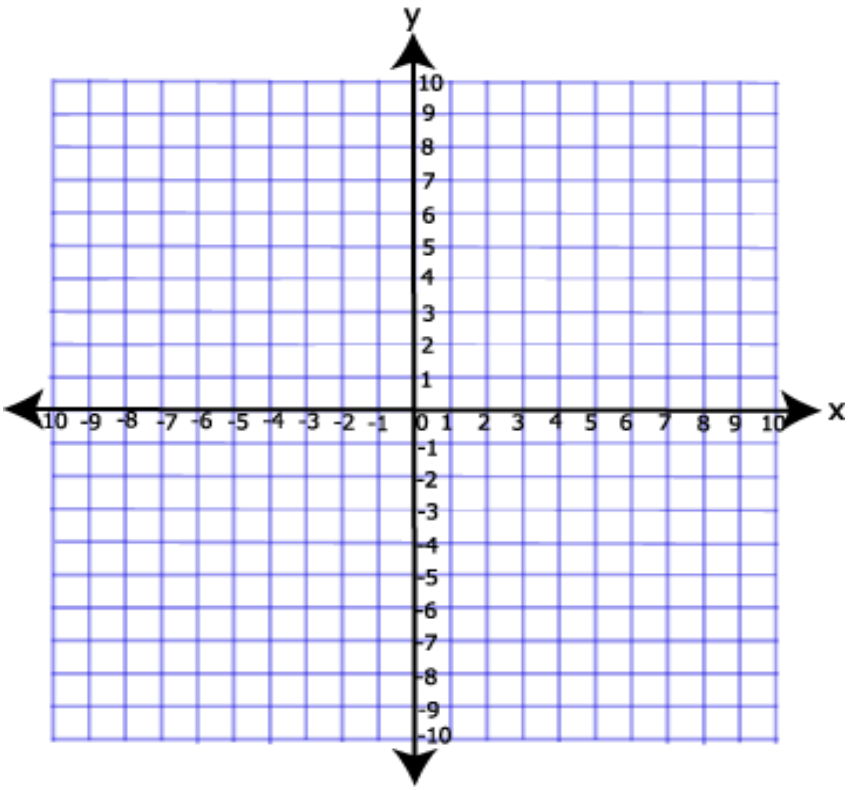
If you are moving RIGHT or LEFT the _____ part of your ordered pair will change. If you go right you will _____ the number of units to the original x. If you go left you will _____ the number of units to the original x.

Graph and label the following points and then translate 3 units left and 2 units up. Label and list your new prime points.

M(5, 8) ⟶ M'(_____,_____)

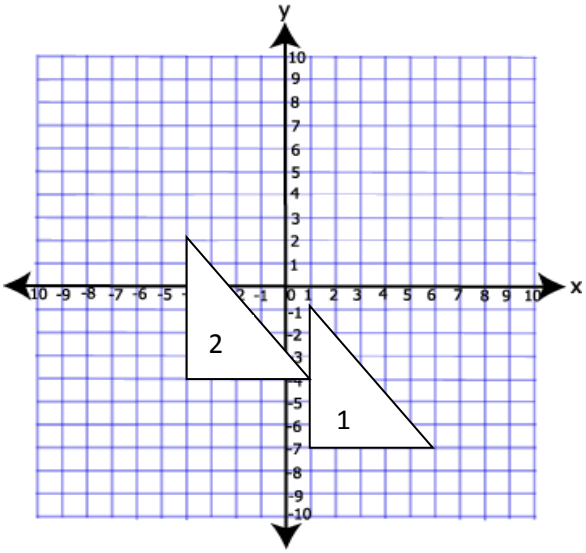
A(0, 6) ⟶ A'(_____,_____)

P(-3, -2) ⟶ P'(_____,_____)



Now describe what happened to each part of the ordered pairs:

(x _____, y_____)



Now describe what happened to each part of the ordered pairs:

(x _____, y_____)

Give the new prime points without creating the graphs for these two translations

Example 1: Translate 2 units left and 4 units down

A(5, -2)	(5_____, -2_____)	A'(_____,_____)
M(2, 6)	(2_____, 6_____)	M'(_____,_____)
B(0, -3)	(0_____,-3_____)	B'(_____,_____)

Example 2: Translate 5 units right and 3 units down

C(3, 5)	(3_____, 5_____)	C'(_____,_____)
W(-2,5)	(-2_____, 5_____)	W'(_____,_____)
T(1, -7)	(1_____,-7_____)	T'(_____,_____)

Translation Practice

1) If point T was located at (4,6) and was moved -3 in the x-direction and -2 in the y-direction, at what coordinates will point T be located?
T'(,)

2) Translate point G located at (-4, 0) +5 in the x-direction and +3 in the y-direction.
G' (,)

3) A point was located at (-3,-2) and was located at (4,5) after it was translated. How many units was it translated in the x-direction and the y-direction?
x-direction_____ y-direction_____

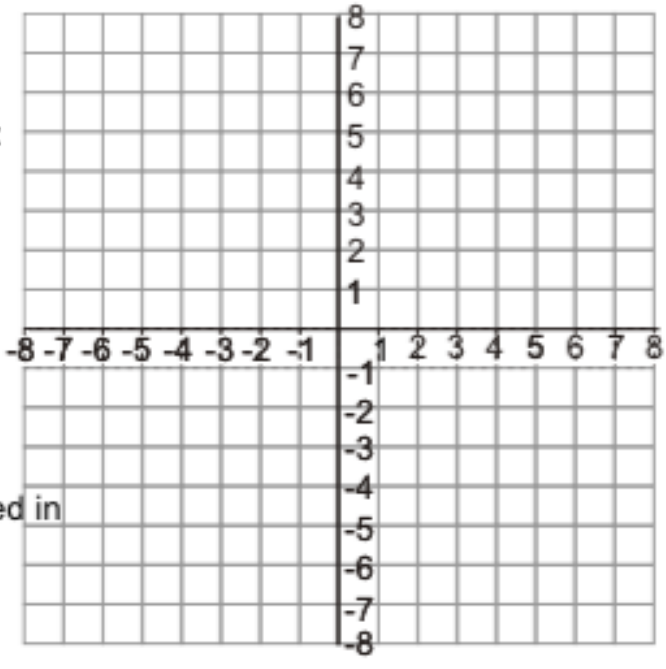
4) If a point was located at (-4,5), how many units would it have to be translated in the x and y direction for the point to reach the origin?
x-direction_____ y-direction_____

5) If point L was located at the origin and was translated +8 in the x-direction and -5 in the y-direction, what would be the new coordinates of point L? L'(,)

6) Translate point R located at (-6, -8) -4 in the x-direction and -2 in the y-direction.
R' (,)

7) Translate point D located at (6, -8) -7 in the x-direction and +5 in the y-direction.
D' (,)

8) Plot and label the following coordinates:
A (-5,1) B(-5,3) C (-1,3) D(-1,1)



9) Translate the object you plotted in the previous problem +6 in the x-direction and -4 in the y-direction.
A'(,)B'(,)C'(,)D'(,)

10) What is the area of the figure you plotted in number 8?_____

Reflections

<u>Pre-Image</u>: The original figure before a transformation

<u>Image</u>: The new figure after a transformation

***Once a point is moved to its new position it is called a "prime point" and named like this: A' (A Prime)**

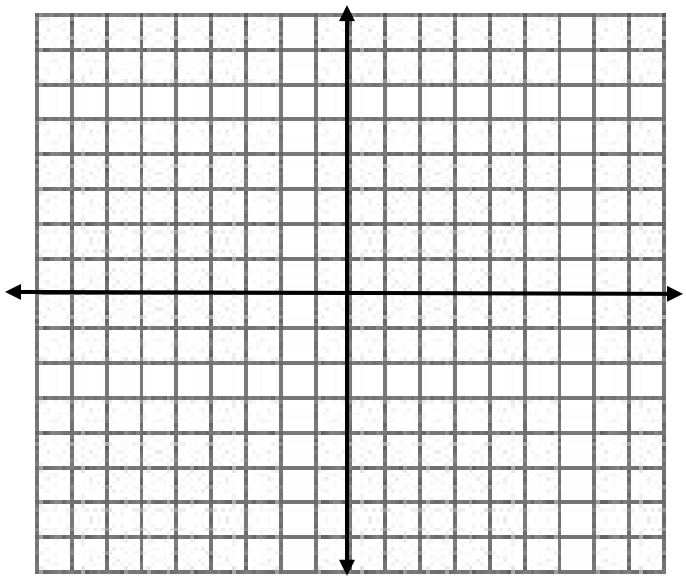
<u>Reflections:</u> Involves move that 'flip' over a given line. The image and pre-image should be symmetrical on both sides of what is called a 'line of reflection' or 'line of symmetry'.

The original shape does not change in size or shape but it does move to a new position.

Example 1:

STEP 1: Create a pre-image by graphing, labeling, and connecting the following points:

A(-3, 2), B(-3, 6), C(-7, 2)



STEP 2: Using a piece of patty paper, and a straight edge, trace the original figure.

STEP 3: FLIP the piece of patty paper over, lining up $\overline{AB}$ an equal distance from the y-axis but on the opposite side of the y-axis.

STEP 4: Record the new coordinates below after the flip and then add this figure to the graph .

A(-3, 2) ⟶ A' (____,____)

B(-3, 6) ⟶ B' (____,____)

C(-7, 2) ⟶ C' (____,____)

Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates after the move. What do you notice?

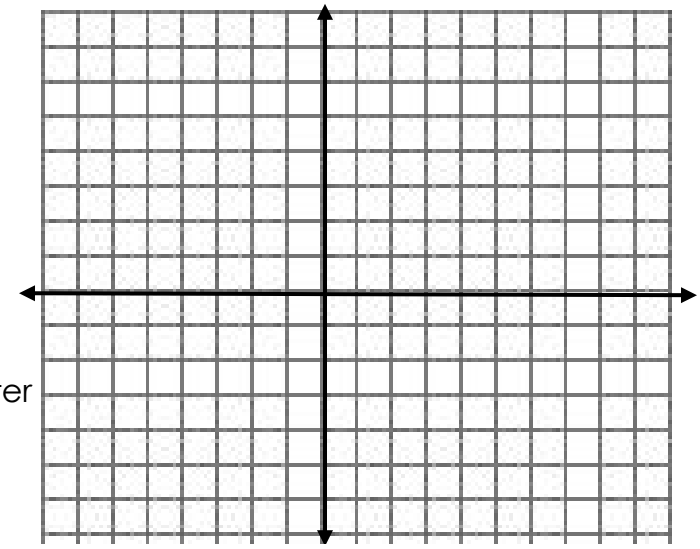
_____

_____

Example 2:

STEP 1: Create a pre-image by graphing, labeling, and connecting the following points:

R(3, 2), T(3, 6), N(8, 1), Q(8, 8)



STEP 2: Using a piece of patty paper, and a straight edge, trace the original figure.

STEP 3: FLIP the piece of patty paper over, lining up $\overline{RT}$ an equal distance from the y-axis but on the opposite side of the y-axis.

STEP 4: Record the new coordinates below after the flip and then add this figure to the graph.

R(3, 2) ⟶ R' (____,____)

T(3, 6) ⟶ T' (____,____)

N(8, 1) ⟶ N' (____,____)

Q(8, 8) ⟶ Q' (____,____)

Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates after the move. What do you notice?

_____

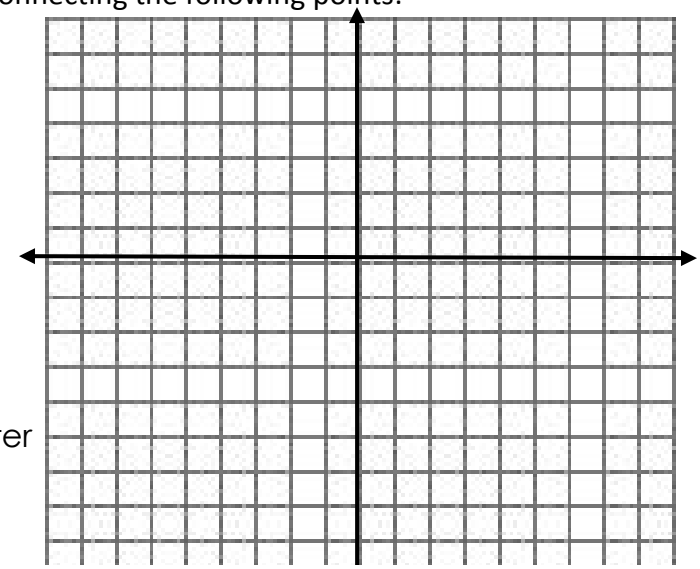
_____

DISCOVERY: When a figure is reflected over the y-axis, the y part of each coordinate _____ and the x part of each coordinate _____.

Example 3:

STEP 1: Create a pre-image by graphing, labeling, and connecting the following points:

A(-3, 2), B(-3, 6), C(-7, 2)



STEP 2: Using a piece of patty paper, and a straight edge, trace the original figure.

STEP 3: FLIP the piece of patty paper over, lining up $\overline{AB}$ an equal distance from the x-axis but on the opposite side of the x-axis.

STEP 4: Record the new coordinates below after the flip and then add this figure to the graph.

A(-3, 2) ⟶ A' (____,____)

B(-3, 6) ⟶ B' (____,____)

C(-7, 2) ⟶ C' (____,____)

Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates after the move. What do you notice?

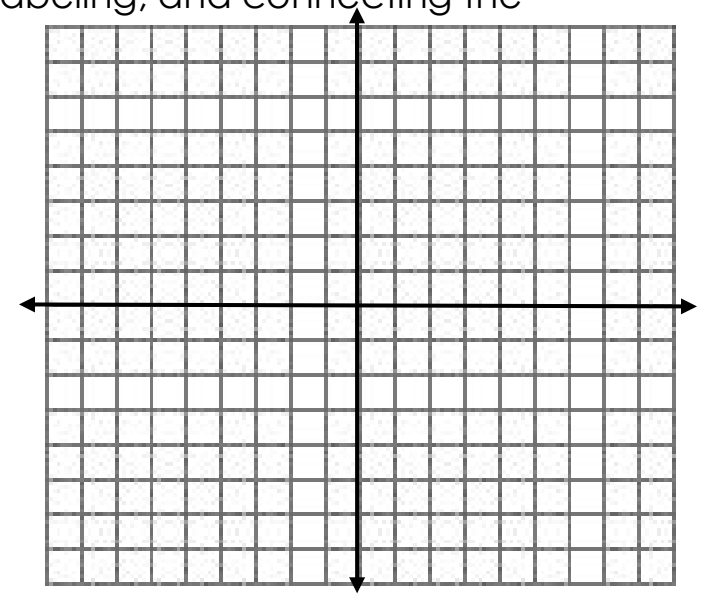
_____

_____

Example 4:

STEP 1: Create a pre-image by graphing, labeling, and connecting the following points:

R(3, 2), T(3, 6), N(8, 1), Q(8, 8)



STEP 2: Using a piece of patty paper, and a straight edge, trace the original figure.

STEP 3: FLIP the piece of patty paper over, lining up $\overline{RT}$ an equal distance from the x-axis but on the opposite side of the x-axis.

STEP 4: Record the new coordinates below after the flip and then add this figure to the graph.

R(3, 2) ⟶ R' (____,____)

T(3, 6) ⟶ T' (____,____)

N(8, 1) ⟶ N' (____,____)

Q(8, 8) ⟶ Q' (____,____)

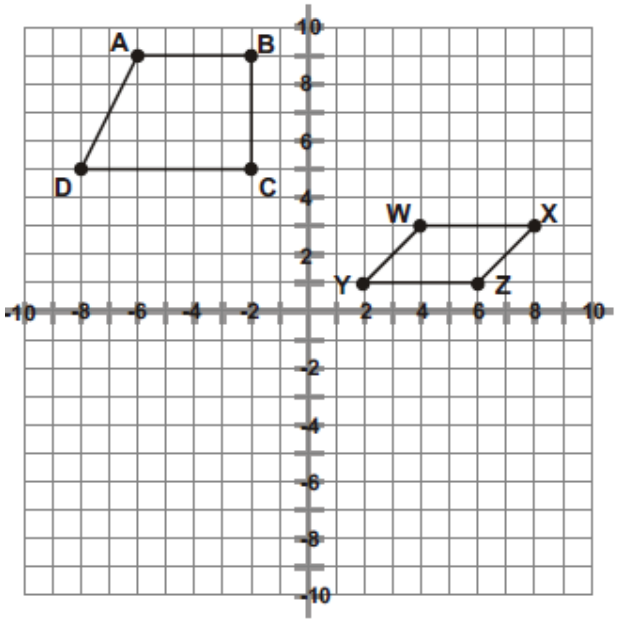
Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates after the move. What do you notice?

_____

_____

DISCOVERY: When a figure is reflected over the x-axis, the x part of each coordinate _____ and the y part of each coordinate _____.

Translate and Reflect – Practice



1) Reflect quadrilateral ABCD across the y-axis. Draw the reflection on the coordinate plane to the right. Record the new location of points A'B'C'D' below:

A' = _____ B' = _____ C' = _____ D' = _____

2) Reflect parallelogram WXYZ across the x-axis. Draw the reflection on the coordinate plane to the right. Record the new location of points W'X'Y'Z' below:

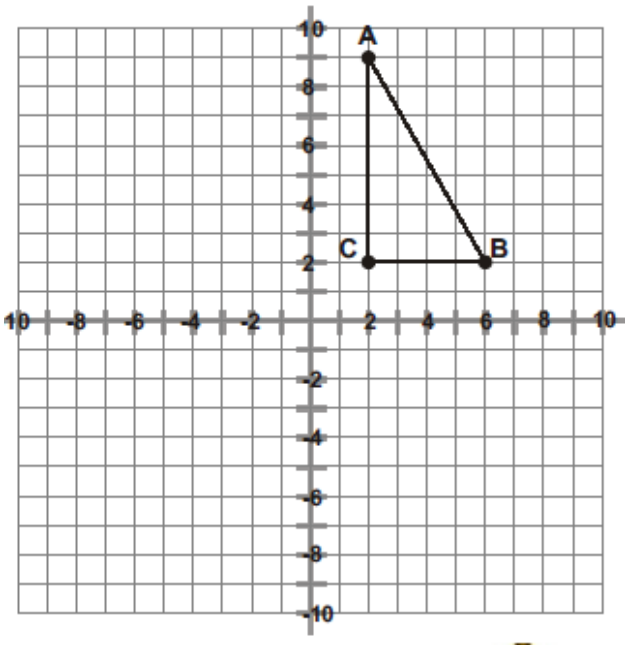
W' = _____ X' = _____ Y' = _____ Z' = _____

3) Translate parallelogram W'X'Y'Z' by sliding it to the left 9 and down 6. Draw the translation on the coordinate plane. Record the new location of points W''X''Y''Z'' below:

W'' = _____ X'' = _____ Y'' = _____ Z'' = _____

4) Translate triangle ABC positive 3 in the x direction and negative 8 in the y direction. Draw the new location of triangle ABC on the coordinate plane. Record the new coordinates for A'B'C' below:

A' = _____ B' = _____ C' = _____



5) Reflect triangle A'B'C' across the y-axis. Draw the triangle on the coordinate plane. Record the new coordinates of triangle A''B''C'' below:

A'' = _____ B'' = _____ C'' = _____

---

6) Reflect figure ABCD across the x-axis. Draw the reflected figure on the coordinate plane. What are the new coordinates?

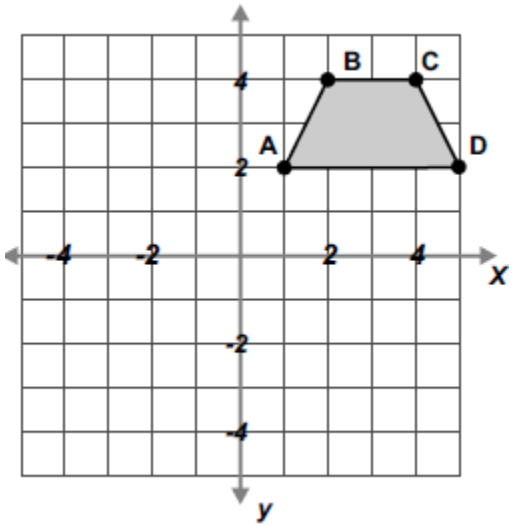
A' = _____ B' = _____ C' = _____ D' = _____

7) Reflect figure ABCD (pre-image) across the y-axis. Draw the reflected figure on the coordinate plane. What are the new coordinates?

A' = _____ B' = _____ C' = _____ D' = _____

8) After reflecting ABCD across the x-axis, what quadrant did the figure end up in?

9) After reflecting ABCD across the y-axis, what quadrant did the figure end up in?



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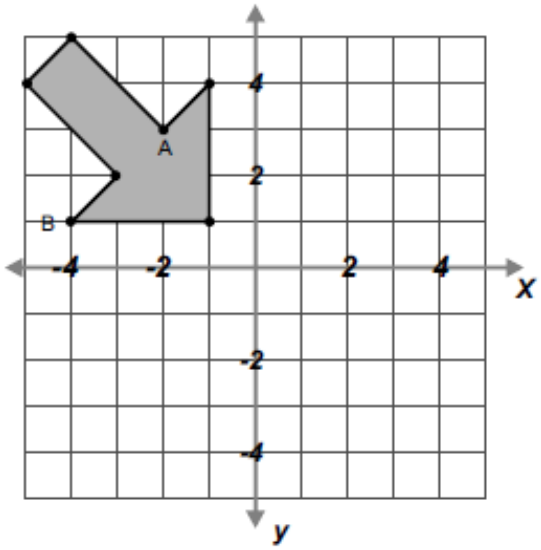
10) If the arrow is reflected across the y-axis, what will the new coordinates be of point A and B?

A' = _____ B' = _____

11) If the arrow is reflected across the x-axis, what will the new coordinates be of point A and B?

A' = _____ B' = _____



Rotations

<u>Pre-Image</u>: The original figure before a transformation

<u>Image</u>: The new figure after a transformation

***Once a point is moved to its new position it is called a "prime point" and named like this: A' (A Prime)**

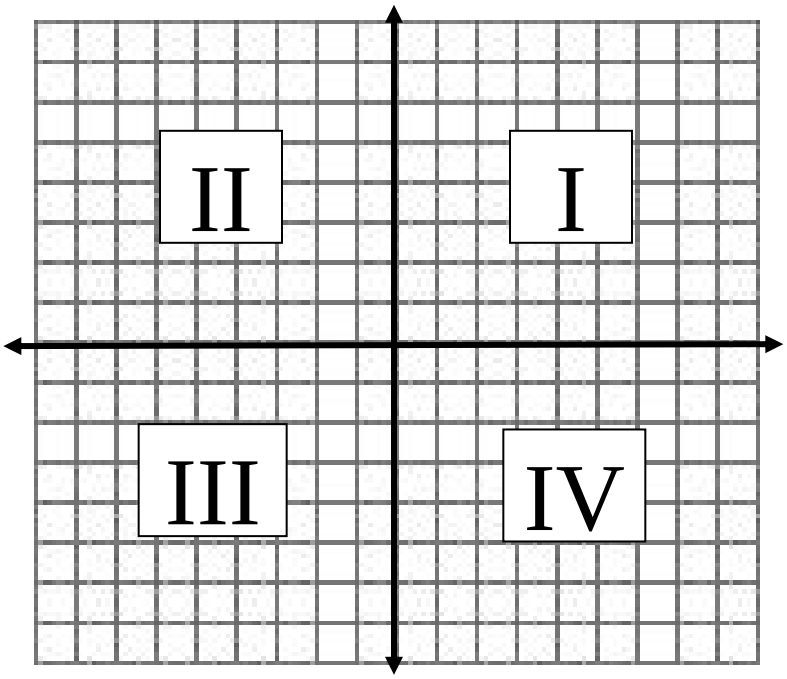
<u>Rotation:</u> A movement of a figure that involves rotating in 90-degree increments around the origin

***The new prime points will be in the quadrant that is the given number of degrees clockwise or counterclockwise from the original figure**

You will need to remember the names of the quadrants: Write whether the x and y coordinates will be positive or negative for each quadrant.

X= ----------

Y=----------



X= ----------

Y=----------

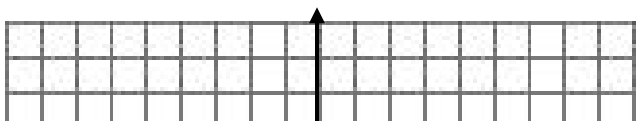
X= ----------

Y=----------

X= ----------

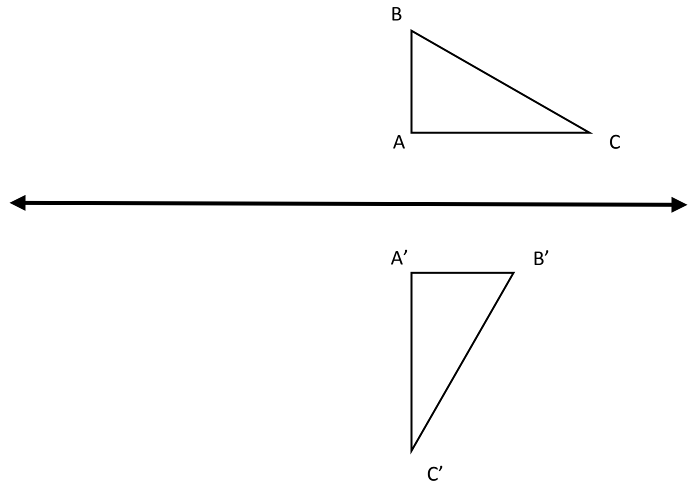
Y=----------

EXAMPLE #1:



STEP 1: The following is a 90^0 clockwise rotation:

STEP 2: List the pre-image points and the image points below.



A(____,____) ⟶ **A' (____,____)**

B(____,____) ⟶ **B' (____,____)**

C(____,____) ⟶ **C' (____,____)**

In what quadrant is the

Pre-Image?______ ; Image?______

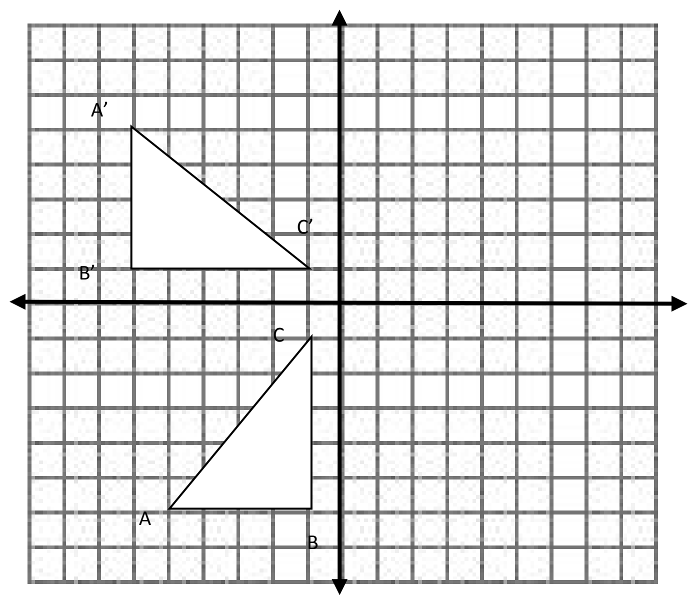
Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates. What is the relationship between the pre-image coordinates and the image coordinates?

____

EXAMPLE #2:

STEP 1: The following is a 90^0 clockwise rotation:

STEP 2: List the pre-image points and the image points below.



A(____,____) ⟶ **A' (____,____)**

B(____,____) ⟶ **B' (____,____)**

C(____,____) ⟶ **C' (____,____)**

In what quadrant is the

Pre-Image?______ ; Image?______

Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates . What is the relationship between the pre-image coordinates and the image coordinates?

____

Using the two examples above, describe what happens to the coordinates in a 90^0 clockwise rotation?

____

Using the rule you have discovered, find the prime coordinates for a line with pre-image points at (2, -6) ⟶ (____,____)

(7, -1) ⟶ (____,____)

EXAMPLE #3:

STEP 1: The following is a 90^0 counter-clockwise rotation:

STEP 2: List the pre-image points and the image points below.

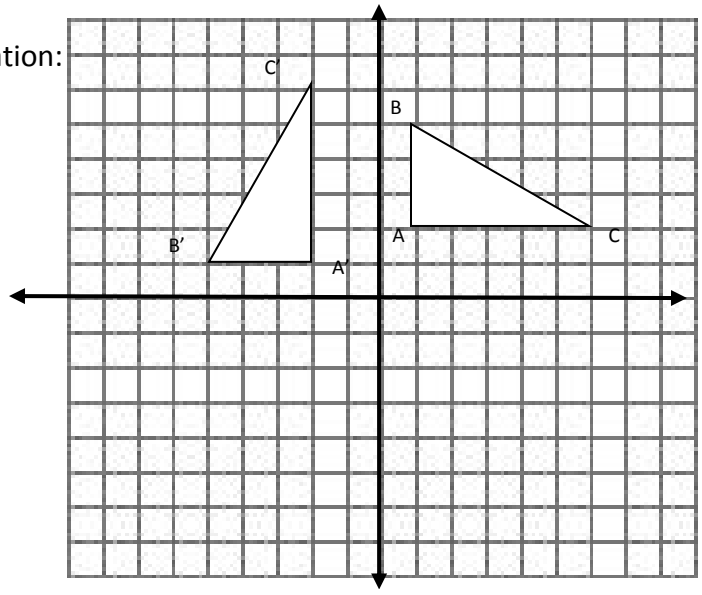
A(____,____) ⟶ A' (____,____)

B(____,____) ⟶ B' (____,____)

C(____,____) ⟶ C' (____,____)

In what quadrant is the

Pre-Image?_____ ; Image?_____



Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates. What is the relationship between the pre-image coordinates and the image coordinates?

_____

EXAMPLE #4:

STEP 1: The following is a 90^0 counter-clockwise rotation:

STEP 2: List the pre-image points and the image points below.

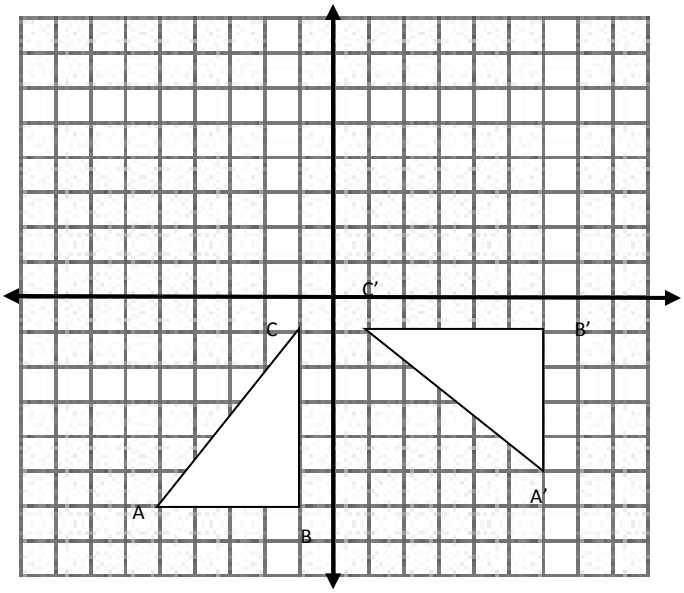
A(____,____) ⟶ A' (____,____)

B(____,____) ⟶ B' (____,____)

C(____,____) ⟶ C' (____,____)

In what quadrant is the

Pre-Image?_____ ; Image?_____



Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates . What is the relationship between the pre-image coordinates and the image coordinates?

_____

Using the two examples above, describe what happens to the coordinates in a 90^0 counter-clockwise rotation?

_____

Using the rule you have discovered, find the prime coordinates for a line with pre-image points at (2, -6) ⟶ (____,____)

(7, -1) ⟶ (____,____)

EXAMPLE #5:

STEP 1: The following is a 180^0 rotation:

STEP 2: List the pre-image points and the image points below.

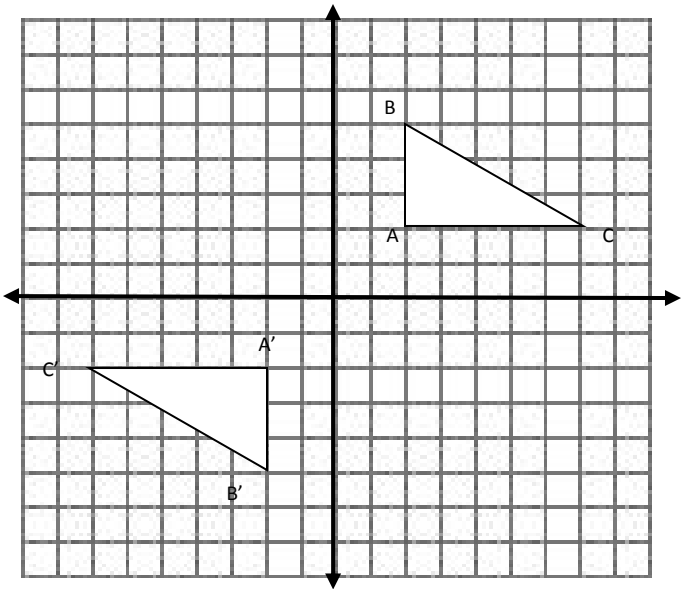
A(____,____) ⟶ A' (____,____)

B(____,____) ⟶ B' (____,____)

C(____,____) ⟶ C' (____,____)

In what quadrant is the

Pre-Image?_____ ; Image?_____



Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates. What is the relationship between the pre-image coordinates and the image coordinates?

_____

EXAMPLE #6:

STEP 1: The following is a 180^0 clockwise rotation:

STEP 2: List the pre-image points and the image points below.

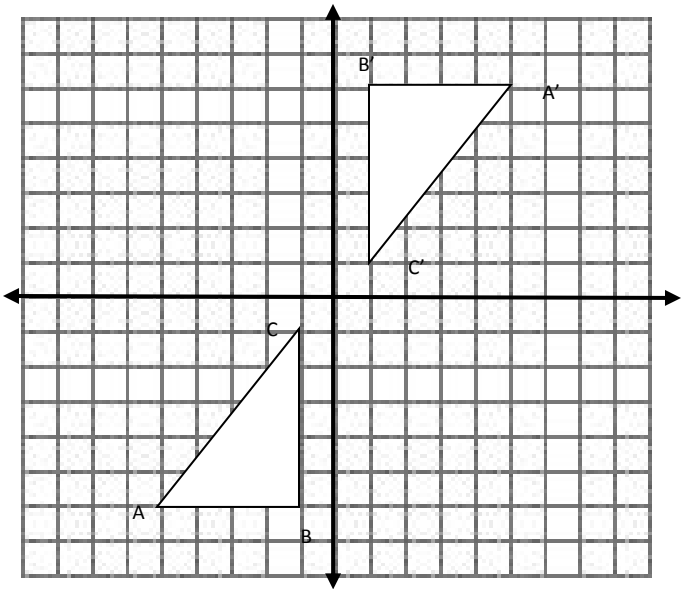
A(____,____) ⟶ A' (____,____)

B(____,____) ⟶ B' (____,____)

C(____,____) ⟶ C' (____,____)

In what quadrant is the

Pre-Image?_____ ; Image?_____



Compare each set of PRE-IMAGE coordinates with the IMAGE coordinates . What is the relationship between the pre-image coordinates and the image coordinates?

_____

Using the two examples above, describe what happens to the coordinates in a 180^0 rotation?

_____

Using the rule you have discovered, find the prime coordinates for a line with pre-image points at (2, -6) ⟶ (____,____)

(7, -1) ⟶ (____,____)

Dilations

Pre-Image: The original figure before a transformation

Image: The new figure after a transformation

***Once a point is moved to its new position it is called a "prime point" and named like this: A' (A Prime)**

Dilations: involves enlarging or reducing the size of an object. Dilated figures are always similar to each other since the sizes get multiplied or divided.

Scale Factor: will determine how much larger or smaller an object will become.

- **A scale factor greater than one = increase in size**
- **A scale factor less than one = decrease in size**

Scale Factor: Looking at the coordinates below, identify what scale factor was used. (Hint) Compare how much of an increase or decrease there was between x coordinates and y coordinates.

EXAMPLE 1:

A (5, 3), B(2, 5)

A'(10, 6), B'(4, 10)

SCALE FACTOR:_____

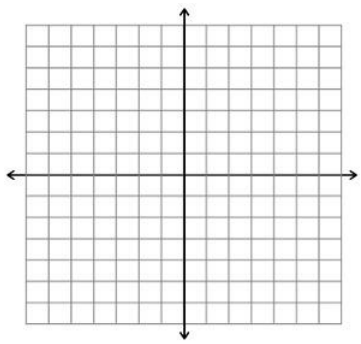
EXAMPLE 2:

X(-2, 4), Y(2, 8)

X'(-1, 2), Y'(1, 4)

SCALE FACTOR:_____

From example one, plot points for X and X'. Then draw a straight line connecting them. Extend the line and see if it goes through the origin?



EXAMPLE 3:

Plot the following points for a triangle and then draw the triangle. Next apply a scale factor of 3. Write the new prime points below and then plot them on the provided graph.

R(1, 1) ⟶ R'(____, _____)

S(1, 4) ⟶ S'(____, _____)

T(4, 1) ⟶ T'(____, _____)

What happens if you go to the graphed figures and draw lines extended from each given point and its prime?

How could you tell that the graphed triangle would get larger or smaller?

EXAMPLE 4:

Plot the following points and draw the figure. Next apply a scale factor of 1/3. Write the new prime points below and then plot and draw them on the provided graph.

M(-3, -3) ⟶ M'(____,____)

N(0, -9) ⟶ N'(____,____)

Q(-9, -3) ⟶ Q'(____,____)

D(-12, -9) ⟶ D'(____,____)

What happens if you go to the graphed figures and draw lines extended from each given point and its prime?

How could you tell that the graphed triangle would get larger or smaller?

EXAMPLE 5:

What do you think may happen if the scale factor is negative? _____

_____Try it with this figure using a scale factor of -2:

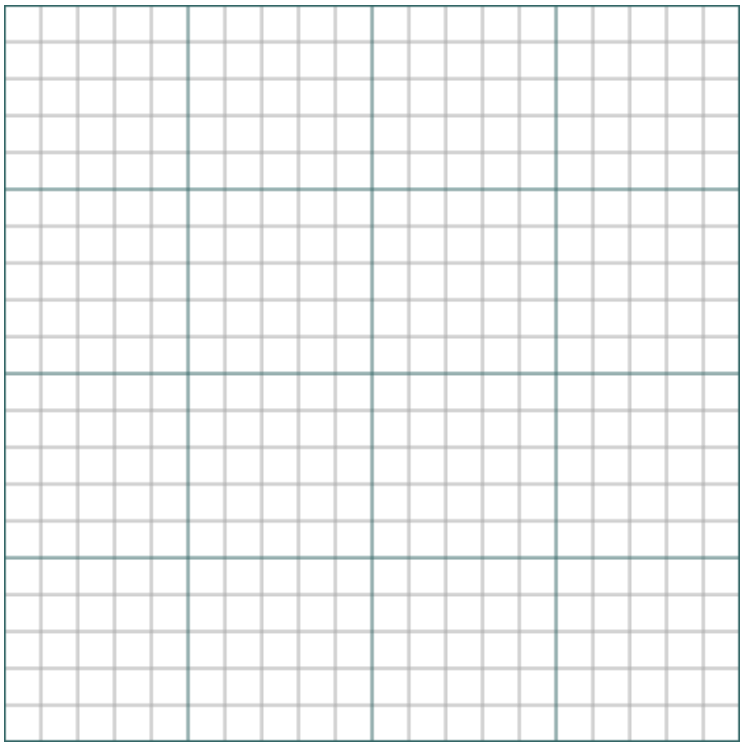
D(-2, 1) D'(____,____)

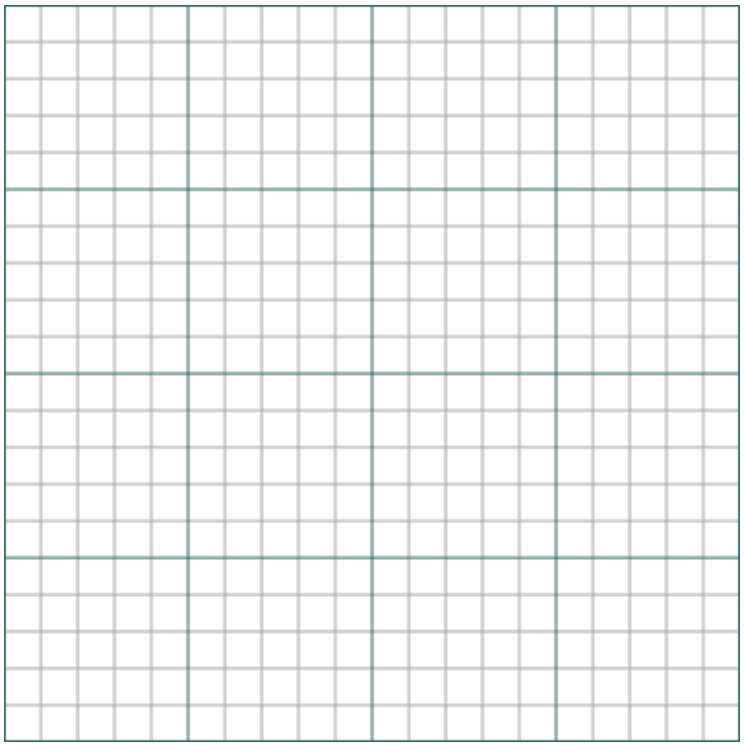
A(-5, 1) A'(____,____)

B(-2, 6) B'(____,____)

What happens with this figure if you go to the graphed figures and draw lines extended from each given point and its prime? Does it still go through the origin?

Describe what happened to the figure:





Dilations Practice

1. Given the following points for an image, list the prime points after a dilation using a scale factor of 3:

W(1, 1), S(2, 1), T(1, 2) G(2, 2) ⟶ W'(___,___), S'(___,___), T'(___,___), G'(___, ___)

2. For a given dilation, the point (5, 0) has a prime point of (35, 0). What are the coordinates of the prime point using the same dilation for (10, 2)? (____, ____)

3. What are the coordinates for the prime point given an original point of (4, 6) after a scale factor of -6 is applied? (____, ____)

4. For the following, identify the scale factor that was used.

D(7, -3) ⟶ D'(14, -6)

E(-2, -5) ⟶ E'(-4, -10) SCALE FACTOR:_____

M(8, 2) ⟶ M'(16, 4)

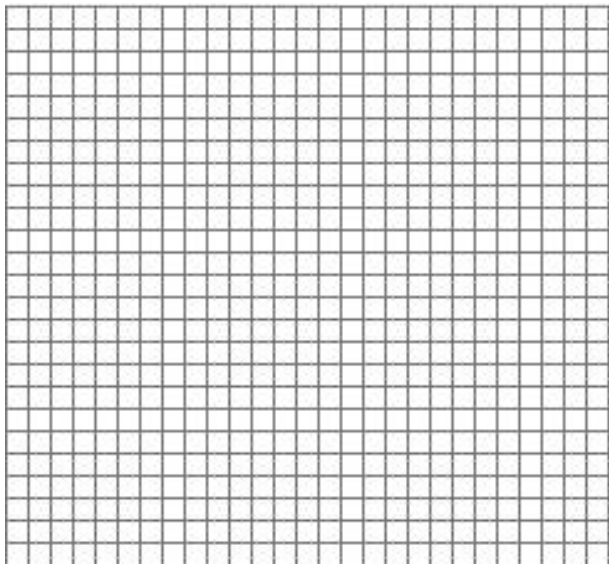
5. Graph the following image then apply a scale factor of ¼ . Draw the new image on the same graph and list the prime points.

E(4, -2) E'(____,____)

X(4, -8) X'(____,____)

A(12, -4) A'(____,____)

L(12, -8) L'(____,____)



6. How do you know if an image will get larger or smaller based on the scale factor?_____

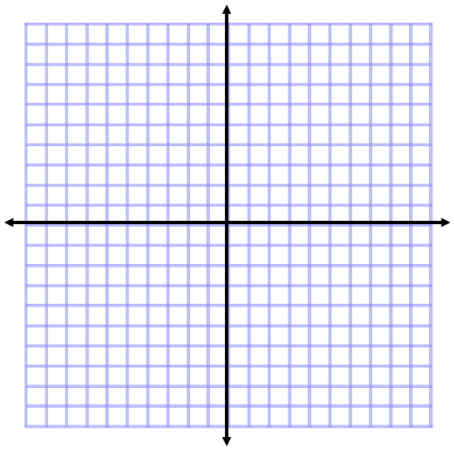
_____

_____

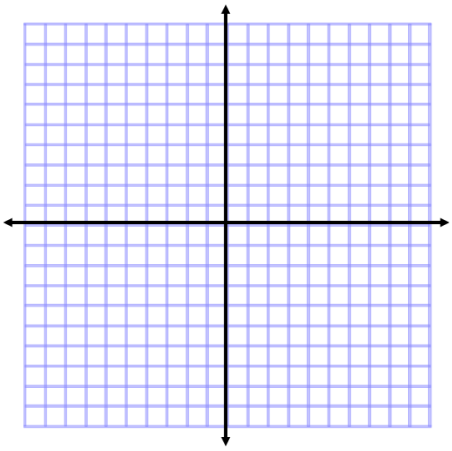
Composition of Transformations

1) Given ΔDEF with D(3, 1), E(-3, 2), and F(-2, -2). Find the image points after:

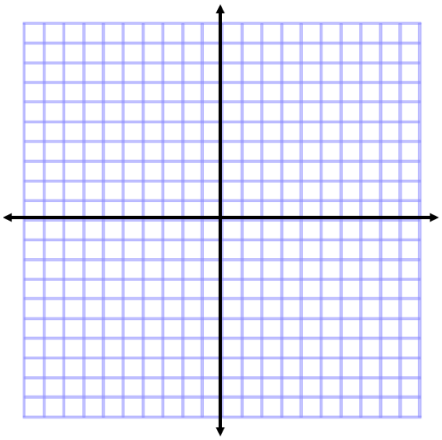
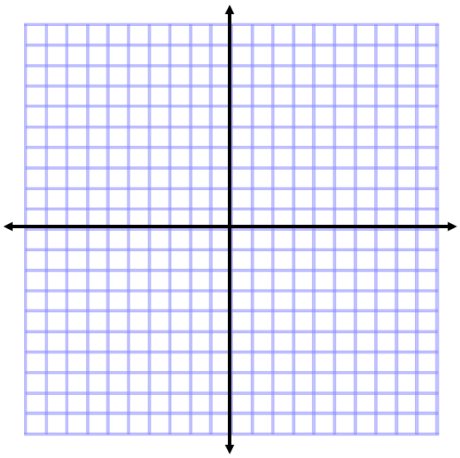
a. FIRST a reflection over the x-axis, SECOND then a dilation of $\frac{1}{3}$.



b. FIRST a translation of (x, y) → (x - 5, y + 2), then a rotation of 90° counter clockwise.



c. A reflection over y = x, then a translation of (x, y) → (x + 1, y – 4).



2) Triangle DEF has vertices D(3, -4), E(2, -2), and F(0, 1). Find the coordinates after a glide reflection composed of the translation (x, y) → (x, y - 2) and a reflection in the y-axis.